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Some elements on everything you always wanted to know about introducing eductive reasoning in negative feedback environments

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Abstract

In this paper we show how is put into practice one particular process of beliefs formation that agents are likely to undertake in order to answer the behavioural uncertainties they face. Within extreme situations, agents' expectations can either be based on a focal point, or on a process of complex reasoning in which they "forecast the forecasts of the others"(Binmore, 1987), i.e. the eductive reasoning; intermediary situations are possible. As this type of reasoning results from the accumulation of different successive steps, it is probable that the process stops before reaching infinitum, either involuntarily, because the agents have limited eductive abilities, or voluntarily, because they impose themselves a limit, estimating that their opponents will have lower limits. We introduce here a discussion on the negative feedback systems: they correspond to situations where two successive actions have opposite consequences; they are strategic substitutabilities situations, where the eductive reasoning is supported and stabilized. We try to asses the rule that determines the stopping rule of the eductive reasoning, corresponding to the moment when the reflective beliefs become intuitive (Sperber, 1997). We show that this moment rises from the architecture of the verbal communication and is thus "natural" in negative feedback situations; moreover, it is the consequence of a "number attitude" (Dehaene, 1993).

Key Words: Focal point, eductive reasoning, negative feedback

1. Introduction

Let imagine a situation in which all agents confronted with an environment hold exactly the same information about it and its functioning at time *t*. Yet within this symmetric informational situation, agents are not identically symmetric because of their different cognitive abilities. Each agent receives information and processes it given his computational capacity and, on the basis of this processing, forms expectations about the state of the world in period *t*+1 or in subsequent periods. An important part of economic research is devoted to address the question about how exactly people use the available information to form their beliefs, deliver predictions and take decisions based on this activity. Whereas first attempts to explain expectations formation attached importance to simple rules as naïve or adaptive expectations[1], two empirical evidences put into light the fact that, first, several forecasting rules should be taken into account[2] and, second, their complexity should increase. Muth (1961), for example, was motivated by the observation that in simple forecasting rules models, systematic errors made by agents are not used to improve future behaviour and agents keep making the same mistakes without learning. He thus proposed to introduce the rational expectations (RE) rule as a more complete expectations rule, which has the advantage that when adopting it, agents do not have incentives to change, whereas under alternative assumptions an individual agent could obtain higher utility by adopting a different rule. This new rule requires common knowledge about the market equations (common understanding of all available information): the complete information set is used by agents to compute their forecasts. Under the RE assumption, agents are supposed not to make systematic errors any longer and to be on average as good as the prediction of the theory[3]. The rational-expectations hypothesis (REH) is thus nothing else than the

[1] *Agents with naive expectations expect today's price to hold tomorrow* ($E_{t-1}x_t=x_{t-1}$)*; agents holding adaptive rules expect tomorrow's price to be a weighted average of the last observed price* x_{t-1} *and the last expected price* $E_{t-2}x_{t-1}$.

[2] *Because different cognitive constraints result in different forecasting rules.*

[3] *"Expectations of firms tend to be distributed, for the same information set, about the prediction of the theory"(Muth, 1961).*

extension of the rationality hypothesis to expectations. In other words, people make the right forecasts because it is in their own interest. Guesnerie (1992) suggests interpreting this assertion dually: it right that it is in the interest of agents to make correct forecasts; it is wrong to assume that perfect coordination is the necessary outcome of an independent optimizing effort of isolated agents. A right forecast must take into account the possibility of wrong forecasts from the other agents. Therefore we should interpret the rational-expectations hypothesis as a *consequence* of rationality and common knowledge of rationality. Nowadays the economic research has come to address the question of the amount of knowledge that is assumed for agents functioning under the RE rules. As agents have different cognitive abilities, which are in addition associated with information processing costs, it is possible that their computational resources are limited and therefore a hiatus must be put on the perfect common knowledge assumed in the REH.

This paper is interested in the manner in which agents understand available information and assimilate it trough reasoning when forming their forecasts in some particular environments that we will describe later: the formation and the articulation of economic beliefs. We focus our attention on beliefs that are formed in these environments and that are rationalizable in the terminology of Bernheim (1984) and Pearce (1984): rationalizable solutions[4] derive from two fundamental principles, i.e. individual Bayesian rationality and common knowledge of this rationality. A specific connection is assumed to hold between decisions and expectations: today's decisions will tomorrow affect the price on which they are based. We need beliefs to be defined within this framework because we are interested in evaluating them with respect to the cognitive profiles that agents exhibit. To put it more clearly, we aim to investigate the reasoning process driving agents to eliminate strategies which would not allow them to maximize their profits (dominated strategies). As in our following studies agents are likely to undertake this type of actions simultaneously, available strategies are continually updated, each agent deploying its respective computational ability in parallel. The processes of elimination of dominated strategies are therefore likely to run several times, because each agent has to take into account the new environment structure (to the extent of his cognitive profile limits) before forming a forecast: thus the reasoning process is iterative. Iterative dominance was introduced in Luce and Raiffa (1957) and studied by Moulin (1979), before Bernheim (1984) and Pearce (1984) reviewed the idea, making the rationalizability popular among economists. Common knowledge was introduced by Lewis (1969) and discussed by Aumann (1976), and made formally clear by Tan and Werlang (1988). Information processing, beliefs formation and finally decision taking imply the exploitation of one's cognitive capacities, i.e. a complex mental activity based on logic operators that is called, since Binomore (1987), *eduction*.

This type of complex reasoning is difficult to put into practice and therefore we will look for some factors which are likely to favour it or, less demanding, to not disturb it; in particular, eductive reasoning could perform better in non-disturbing or stabilizing situations. This paper is therefore focused on the performance of this type of eductive reasoning contingent to a precise structure of the environment in which operate some stabilization factors: we examine eductive abilities of agents in negative feedback situations.

Therefore, this paper is dedicated to the assessment of the eductive type of reasoning in negative feedback situations. Section 2 gives an overview of the process of expectations formation. Section 3 clarifies the mechanism of an eductive-type of reasoning; in section 4 a negative feedback environment is characterized and in section 5 some conclusions are drawn. All along this paper, we try to give a non (or minimalist)-formalized view of research that is relevant on eductive reasoning and negative feedback and to make a way into understanding the *philosophy* of the argument.

2. The process of expectations formation

When people are confronted with a situation, they generally experience uncertainty about the environment itself or about their simultaneous opponents; therefore, there is a need for an agent to form beliefs and expectations. Formally, when the probability of the occurrence of a given event in some situation is invariant to an individual's action, we can say that that particular individual experiences exogenous uncertainty; otherwise, uncertainty is endogenous or behavioural. Keynes (1936)' quotation about the beauty contest is the more famous example in which an individual is put into a situation of behavioural uncertainty that he has to handle and thus is required to anticipate "what average opinion expects the average opinion to be". In economic literature, expectations may be viewed as formalized beliefs and sometimes they are considered as the same concept. By expectations we usually understand as attitudes, dispositions or psychological states of mind that relate to events the outcomes of which are uncertain (Katona, 1951). Three elements are usually involved in an expectation: the individual (who expects), the evidence (what the individual knows) and the prediction (the individual's view of what is going to happen) (Georgescu-Roegen, 1958). The

[4] *Following mainly Bernheim (1984), Pearce (1984) and Guesnerie (1992), rationalizable solutions are conceptually closely related to the successive elimination of dominated strategies.*

process by the means of which the individual is able to transit from the evidence through the prediction depends on the cognitive ability of each agent and of its cost-resources-benefits arbitrage. Prediction can arrive at once (primary expectations) or in steps (higher order expectations).

In conditions where behavioural uncertainty is predominant, when trying to form predictions or expectations, agents are likely to try to detect und understand patterns of regularity and to watch for changes, following their "immensely powerful need for regularity" underlined in Popper (1972). When trying to understand the environment (and especially the others) and its rules of functioning, agents are generally assumed to predict their opponents behaviour using a *theory of mind* (Stennek, 2000; Perner, 1988; Gopnik, 1993). Agents are assumed to make expectations because they help them at reaching the equilibrium; at the equilibrium, agents are coordinated. There are several reasons for which they may have reached coordination, that we present as two polar and two intermediary cases:

i) the equilibrium is a *focal point*; it is *salient* for agents and works as a natural attractor for agents strategies; in this case, the agent doesn't need to put into practice a sophisticated reasoning to select the equilibrium: he just picks the attractor as a strategy that "naturally comes into his mind". This salience situation was described by Schelling (1960) and it corresponds to what is called a first order salience.

ii) agents posses high reasoning capacities and make cross-use of them in order to reach the equilibrium; they put into practice complex reasoning processes which lead them to the equilibrium; this kind of sophisticated reasoning in which agents *forecast the forecast of others,* and understand the logic of the game, is eductive reasoning. Even through we consider eductive reasoning as a polar case, this kind of reasoning may not be complete; as it works by cross-forecast, it can be formalized in steps or stages, and the number of steps or stages that an agent put into practice may not tend to infinitum. When eductive learning is complete, this coordination situation is the one emphasized by Muth (1961) within the REH framework. Notwithstanding, even within a limited number of steps case, it still remains a complex case of thinking, and deep understanding of the model, that will be extensively explained in the following section.

iii) in the absence of a focal point and without important thinking capacities, agents may try to use some *reasonable* rules to reach the equilibrium. In this case, the equilibrium is not salient but, cognitively speaking, agents are of the same type as previous: they may want to select a focal point because they have limited computational abilities and the existence of a natural (visible) attractor is an easy task which corresponds to their ability. But when the equilibrium doesn't exhibit the qualities of an (immediate) focal point, agents have to make use of their limited capacities to come with a strategy and thus they provide strategies which are reasonable in comparison with their thinking capacity. This case corresponds to models of classic bounded rationality, i.e. naïve, adaptive learning, etc.

iv) in the system there might be a focal point, although the agents do not use this first-order salience, but best-respond to first-order salience and may do it repeatedly (*n*-order Shelling salience). At the limit, the equilibrium will be a revealed focal point[5]. This process occurs in steps, but not under the same premises as eductive reasoning. Stennek (2000) defines a hierarchy of increasingly *intelligent* decision-making procedures, which reason analogously to the levels of strict dominance. The bases of this hierarchy are non-rational procedures in which actions are chosen according to their unmodeled salience. Their choices are taken into account in the consecutive higher order procedure, which is rational and chooses an undominated action. Higher order procedures are defined in a similar way until the last infinite-rationality procedure in which iterations are conducted to infinitum.

We indicated in this classification that, on the one hand, there are processes of expectations formation which are simple and depends on limited rationality; on the other hand, there are processes of expectation formation which occur in steps but which are likely to not to be driven ad infinitum. There are differences between expectations processes limits:

i) characteristic limited rationality is involuntary, i.e. agents use proxies instead of a full understanding of the system.

ii) rationality limits are self-imposed when agents are able to jump intermediary reasoning stages and understand the equilibrium.

When an agent ignores the decisions of opponents at the moment of the decision-making, following Bernheim (1984), rationality consists of making a choice which is justifiable by an internally consistent system of beliefs, rather that one which is optimal, *post hoc*. Simon (1982) also considers behaviour as rational when it is the outcome of an appropriate deliberation process. The hypothesis that Nash equilibrium may be directly salient for agents is stronger than the hypothesis that agents attempt to second-guess each other, assuming that their opponents do the same. Bernheim (1984) introduces

[5] *A natural focal point is spontaneously salient; a revealed focal point is reason-salient, i.e. it corresponds to the point of convergence of the* n*-order salience; an eductive focal point is singled out through eductive introspection.*

and justifies a new notion as follows: an agent must construct an assessment on an opponent action and optimize accordingly. This assessment must be consistent with everything the agent knows about the game. Among other things, the agent knows that his opponent has also an assessment in return about what the agent will do for which the opponent's answer is a best response. In order to preserve consistency, the agent must not only have an assessment of what the opponent will do subject to which the agent's choice is a best-response, but for every forecast of the opponent strategy to which the agent assigns positive probability. The agent must also be able to construct a conjecture of the opponent assessment on the agent's action for which this forecast of the opponent strategy is a best response. This reasoning can be extended ad infinitum. If it is possible to justify the choice of a particular strategy by constructing infinite sequences of self-justifying conjectured assessments in this way, than Bernheim (1984) calls the strategy *rationalizable*. But, in practice, agents may only check the consistency of their beliefs for a finite number of levels.

The type of expectations that drives the system determines the kind of equilibrium which is attained. Generally speaking, expectations are based on several closely connected mechanisms: observation, repetition and understanding. When all these processes are complete, the REH paradigm emerges and drives the system to the rational expectations equilibrium (REE). A REE can be characterized by three main features: *i)* market clearing at the equilibrium price; *ii)* the relation between price determination and private information is known by every agent; *iii)* agents fully exploit the information contained in prices. Therefore the REE has a dual role: clearing the market and making public private information.

But Arthur (1994) argues that the level at which humans can apply perfect rationality is surprisingly modest. Therefore, we have to take into account the fact that the REH may be declined and used to several different extents, which are separated by the degree of (intelligent) computations that are run before the decision-making[6]. Therefore the theory of mind is a specific cognitive ability to understand others as reasoning agents, i.e. to interpret their minds in terms of theoretical concepts of intentional states such as beliefs (Davidson, 1980; Gordon, 1986), and to perform mental simulation to the extent of their cognitive *resources*, which gives it more economic flavour. The question is not if agents possess unlimited rationality, but about what approximations of rationality could provide outputs as good as unlimited rationality (how close to the limits are we). Therefore, we will analyse the internal process of the rational expectations process, i.e. the steps involved in eductive reasoning.

Two main questions about expectations are relevant in this paper: why are expectations important and how can we observe expectations. Expectations will be seen as two-dimensioned: agents form expectations about the economic values and interactions and this confers them a strategic dimension; as beliefs, expectations have a cognitive dimension.

Therefore, addressing the first question, expectations are important because they link economics to cognition and behaviour. It is in this sense that Keynes stressed the conventional role of expectations in the financial market activity or speculation; that Hayek analysed the role of information in markets self-organisation; this question was also addressed by Simon who took into account the economic constraints in obtaining and processing information; that Aumann formalized common beliefs pre-coordination role; that Muth stressed an optimal utilization of the information when forming anticipations, and finally Sperber analysed the arbitrage process between the cognitive effort and beliefs pertinence. Experimental economics address the question of how to observe expectations and this leads us to our second question.

In real markets, it is difficult to obtain detailed and unbiased information about agents' expectations. Moreover, we are not able to correctly define the amount of knowledge that agents in real markets have about the market. Experimental economics offers a tool to generate expectations in laboratory. The economy fundamentals and the information that is delivered to subjects are controlled. Experiments allow collecting expectation without additional noise and replicating several times exactly the same environment. Since experimental economics has become an important field in the economics research area, several experimental studies aimed at analysing beliefs and/or expectations. Davis and Holt (1993) and Kagel and Roth (1995) remind the main objectives of an experiment as being theory falsification, sensitivity tests and searching of empirical regularities.

3. Clarifying eductive reasoning

We follow Binmore (1987) and Guesnerie (1992) to first define the concept (3.1), differentiate it from evolutive reasoning (3.2), and establish the links with the common knowledge concept (3.3) and with information processing (3.4)

[6] To point the difference with the classic bounded rationality literature, this can be called bounded or limited REH.

3.1. Definition

Following Binmore (1987), the word *eductive*[7] is used to describe a dynamic process by means of which equilibrium is achieved through careful reasoning on the part of the players before and during the play of the game. Eductive reasoning is therefore a part of a rational decision process and describes the entire reasoning activity that intervenes between the receipt of a decision stimulus and the ultimate decision, especially including the manner in which the agent forms the beliefs on which the decision will be based: this is the mental activity involved in *forecasting the forecast of others*. Such an approach forces this type of reasoning to occur in *steps*; as steps imply oscillations, Binmore (1987) suggests that notions as equilibration or adjustment process should better be understood under the designation *libration*[8].

The eductive reasoning concerns the manner in which the subjects process data, i.e. model their reasoning processes by inputting data in the reasoning processes. Such considerations are internal to a player. The dynamics of an eductive reasoning and other data on the internal thinking process that another co-player may use are not observable; therefore, as emphasized by Binmore (1987) and Guesnerie (1992), such reasoning usually requires an attempt to simulate the reasoning processes of the other players. Eductive reasoning implies careful cross-thinking, symbolized by the lines "if I think that he thinks that I think…"; but this premise requires that the information be available on how an opponent thinks. If real data doesn't exist, performing simulations on other players' minds could be done by introspection, which can be a practical method for predicting the behaviour of others. More specifically, in order to predict what an opponent will do in a situation which cannot be observed, an agent uses what it would do itself, if it were in the same situation, as a guideline. But introspection is not as simple; in the following we will refer to introspection as the process in which an agent incorporates in his own reasoning behaviour elements from other individuals programs, about which he learns to have been successful, and deletes those elements of its original thinking program which are less successful. In the long run, the tendency will be to generate through imitation and education a population with closely similar thinking programs and hence its members will have good reason to suppose that introspection is a valuable source of information about the thinking process of others.

A player endowed with the capacity to put into practice an eductive-type of reasoning can forecast the forecast of others and simultaneously participate in the same game. Being able to introspect while the decision is taken requires that the internal complexity of agents experiencing eductive reasoning is large compared with that of the environment[9]. For the needs of the simulation, perfectly rational players can perfectly duplicate the reasoning process of their opponents and hence perfectly predict their strategies. In an eductive setting, a player needs to be able to state complex hypotheses about the reasoning processes of the opponents. Therefore, to the usual issue of modelling the others as being the same as his own, eductive behaviour must include the possibility of bad play by the opponents and the capacity for an agent to exploit it.

Describing the functioning of the eductive reasoning implies describing and following calculation steps that have to be taken into account when the process is put into practice. The system is driven by a computing apparatus endowed with storage space for all elements it may use in calculating. During the calculation, intermediary results are kept in this working memory. What happens at any step in the calculation depends on the internal state of the agent and of inputs. These factors determine how the agent will calculate and especially the next internal intermediary step to which the agent moves. Based on the rationalizability concept, eductive reasoning steps can be described as following[10]:

i) each agent is rational; he only uses strategies that are best responses to some possible profile of strategies that can be actually played by the others; hence, non best-response strategies are eliminated from the strategy space;

ii) each agent knows that all other agents are rational and imagine a hypothetical distribution of agent types, in which he defines a proportion of agents that are able to perform the previous step; thus several non-best responses to this state are eliminated from the initial strategies set;

iii) another proportion of players are able to perform the previous step and this is commonly known.

(p) all agents know that all agents know that all agents know….that agents have performed the previous steps.

Why it is important is to define a stop procedure, i.e. an agent performing eductive reasoning must finish its calculations at some step which corresponds to his cognitive limit of to one more step over what he expects of his opponents. Stopping at some step doesn't mean eductive reasoning regression is finite, but simply that agents are able to save their efforts by

[7] *To educe = to bring, to draw out, develop, extract or evolve from latent of potential existence; infer a number, a principle, from data or from another state in which it previously existed (from the Latin word* educere*, lead) (Oxford English Dictionary)*

[8] *Libration = very slow oscillation (from the Latin word* libra*, scale) (Oxford English Dictionary)*

[9] *Simon(1955, 1959, 1977) describes agents for which internal complexity is low.*

[10] *Adapted from a specific example in Guesnerie (1992).*

only providing the number of steps that is contingent with the situation and their beliefs about opponents. The stopping rule appears as a rule of thumb for determining when convergence has been sufficiently approached, i.e. when the estimated (small) cost of moving one more step outweighs the estimated benefits of a more refined prediction.

A rational agent cannot be seen as a unique type. An agent is assumed to recognize *n* different possible types of agents, distinguished by the way in which the objective data that they may have received has been processed. The existence of different types of agents performing eductive reasoning implies that several stopping rules exists, therefore the eductive is likely to be interrupted at different moments. Not pushing an eductive process until its end can be considered as a deviation from perfect play. This leaves the possibility of explaining deviations from predicted play without the necessity of abandoning the hypothesis that the opponent is rational. Even if the trembling hand provides similar possibilities, in an eductive context these explanations have to be of last resort (Binomore, 1987).

As real life situations always involve some explicit or implicit constraints on the cost of an action, cost of calculation must be taken into account when describing the eductive process and the stop device. A possibility that had been evoked is to hypothesize meta-players who design the players that actually play (Megiddo and Wigderson, 1986; Neyman, 1985; Rubinstein, 1985; Abreu and Rubinstein, 1986): these meta-players are seen as playing a meta-game in which a pure strategy is the choice of players. But of course this approach only transfers the problem. The example given in Binmore (1987) is the examination of the status of an auctioneer in the Arrow-Debreu model of a market. Except in rare circumstances, such an auctioneer doesn't exist. Calculating prices instead is achieved through an unmodeled tâtonnement process for which the auctioneer serves as a simplifying substitute.

Usually, theoretical realizations of bounded rationality incorporate a fixed, exogenously determined, upper bound on some aspect of the complexity of the strategies available to a player. Bounded rationality in this sense is not costly rationality as far as in an eductive context, a minimal requirement is that the marginal cost of calculation must always be very small. Thus, if a player fails to carry out certain computational tasks in equilibrium, it is because the agent has chosen not to do so, not because it is unable to do so without abandoning other computational tasks. In an eductive context, any computational constraint on an agent must therefore be endogenous (self-imposed). A player must be able to recognize and respond to (expected) deviating behaviour.

3.2. Distinction eductive/ evolutive reasoning

As a strictly eductive environment is seldom encountered in the real world, evolutive factors always contaminate the analysis of eductive settings. The word *evolutive* describes a dynamic process by means of which equilibrium is achieved through evolutionary mechanisms. Binmore (1987) includes in this type of reasoning all very long-run processes studied by evolutionary biologists (Smith, 1982), but also medium-run processes in which the population dynamics are not necessarily based on genetic considerations (Friedman and Rosenthal, 1984), as well as very short-run processes by means of which markets achieve clearing prices (Marschak and Selten, 1978; Moulin, 1981). The linking consideration is that adjustment takes place as a result of iterated play by myopic players.

The dynamics of an evolutive process is external and visible to the observer. It sometime occurs in the same sequence of steps as eductive reasoning. Therefore, the distinction between an eductive (mental) and an evolutive (statistical) process is quantitative rather than qualitative. In the former, players are envisaged as having potentially very high complexity (with low operating costs) whereas, in the latter, their internal complexity is low. However, only isolated forays have so far been made into this area (Neyman, 1985; Rubinstein, 1985; Abreu and Rubinstein, 1986).

Through both processes, the situation evolves. In an eductive setting, evolution (as seen from the exterior) operates at one remove, because notional time cannot be observed; in evolutive processes, evolution works in real time. Strategies of one specific game are directly influenced and evolve in evolutive theory. When costly rationality is taken into account, this means that the evolutionary process act directly on the rules of behaviour which implement these strategies. In eductive theory, on the other hand, evolutionary processes work on a meta-program which has the capacity to choose strategies in a wide variety of different games, some of which may have been never played before.

3.3. From common knowledge to eductive reasoning

In order to handle interactive situations, agents must have not only individual and independent knowledge of the fundamentals (which is usually called first order knowledge), but also *interactive* knowledge (higher order knowledge), i.e. knowledge about others' knowledge. When acting in such situations, agents' behaviour is significantly influenced by their knowledge about the others' knowledge; moreover, by the knowledge they have about others' knowledge about their knowledge, and so on. When such reasoning is applied infinitely and leads to identical knowledge for every agent, this kind of knowledge is called common knowledge (CK). To put it more precisely, a knowledge is common among a group of agents if everyone has it, everyone knows that everyone has it, everyone knows that everyone knows that everyone has it, and so on ad infinitum. Actually, it is simply an infinite regress of reasoning about agents' knowledge which unifies the set of distributed knowledge. It makes collective knowledge completely transparent to each individual.

As this notion is constructed in steps, it can be characterized by the numbers of steps which have been accomplished, i.e. the depth of knowledge. Morris and al. (1995) introduce a formal definition of the depth of knowledge that we adapt here to describe agents' insertion in the informational structure of an environment. Confronted to an environment in which information about fundamentals is symmetrically delivered, agents naturally split into types according to the extent to which each agent is able to understand or to process information. Thus, we transfer the depth of knowledge notion from the environment to an individual and say about an agent that he has a *k*-depth of knowledge if he able to understand the CK structure of the environment until iteration *k*. Therefore, an agent endowed with depth of knowledge *k* is supposed to have accomplished *k* iterations of knowledge. Attaining one additional *unit* of depth of knowledge requires implementing a step of eductive reasoning. Therefore, an agent holding extended[11] knowledge depth about market equations is supposed to have accomplished *k* steps of eductive reasoning. An agent reaches his *type* by eductive reasoning or introspection on the information he receives.

The truth of the assertion (everybody knows)^N (Guesnerie, 2004) that the agents are rational, whatever *N*, defines common knowledge of rationality. Hence, as defined, common knowledge of rationality (and of the game) implies that all agents have *perfect foresight*: the equilibrium is guessed or educed through the process just described; it may be said eductively stable or strongly rational.

3.4. Information processing costs and eductive reasoning

When describing eductive reasoning, we only are interested in contests. Binmore(1987) uses this term to indicate a game in which no pre-play communication between the players is allowed. But the impossibility for players to communicate before and during the play doesn't imply that they do not share information that is supplied to them in accordance with the rules of the game. Among other things, they will share knowledge about conventions involved in the population. Such conventions allow players to coordinate their behaviour by making use of the fact that they can commonly observe phenomena which are not intrinsic to the game, i.e. common understanding. When two rational players face each other in a game, their choices of strategies will be approximately optimal given their predictions of the strategies to be chosen by the other player. But this prediction need not always be realized because of indeterminacy. Therefore, when computing an answer in a game, an agent must previously identify his opponent as belonging to the same population. Players are satisficers in the sense that they are will calculate only to the extent of an appropriate level of approximation.

The assumption of null informational cost is unrealistic. Whenever understanding (by processing) information is costly, an agent endowed with rationality face the decision problem of whether the expected benefit of acquiring or processing the information is worth the cost of processing. Therefore the amount of information processed by individuals becomes an element of the decision making process. When full rationality is scarce, the deliberation cost must be taken into account (Conlisk, 1996) because good decisions are costly. There is a trade-off between effort devoted to deliberation and possible outcome, depicted by Day (1993) as the *economy of the mind*. Establishing a deliberation model was the purpose of research by Marshak and Radner (1972), Selten (1978), Radner and Rothchild (1975), Evans and Ramey (1992,1995). These models generally show how the deliberation technique can merge optimization, rational expectations, and the degree of rationality of a decision is endogenously determined by economic forces. Tirole (2002) describes a simple situation with a three-period horizon (t = 0,1,2). At t = 0, the agent receives information and has a prior on the situation. At time t = 1, he will decide whether to exert some introspection effort at a positive cost or not. Effort results in a probability of success which can be interpreted as ability in the task. When deciding to exert effort, the agent perceives (immediate) current cost to be compared with discounted expected benefits from effort. Therefore ability and effort are sometimes complements, sometimes substitutes in generating outcomes.

4. Beliefs retroaction into the system: the case of negative feedback

Besides individual cognitive processes that agents put into practice in decision making situations, evidence especially on financial markets shows that collective (Orlean, 2000) or environment-based dynamics during which agents expectations and representations are transformed under the action of the others and of the market itself. We therefore have to assume that any model of (rational) reasoning will deliver different performances indifferent environments. In particular, a reasoning process is expected to better

[11] *By extended knowledge we understand knowledge about the environment with all its* mathematical *implications, i.e. the extension of knowledge to rationality.*

perform if no external forces disturb it, or especially in a stabilizing framework. A distinction can be envisaged between confirming environments (in which only the direction indicated in the reasoning process is confirmed) and stabilizing environments (in which the localization of the equilibrium is educed). Confirming environments are viewed in the literature as positive feedback or strategic complementarities environments and stabilizing environments are represented by negative feedback or strategic substitutabilities environments. As the process under scrutiny here is eductive reasoning, which is a rather complex process, we will investigate it in negative environments, which are likely to favour it. Section 1.4.1 will describe such environments; section 1.4.2 will assesses stability criteria in such environments; sections 1.4.3 and 1.4.4 are concerned respectively with intuition and reflection in negative feedback environments and with number attitude.

4.1. A simple description of negative feedback

Following Arthur's (1989) work, more and more attention has been devoted in business and economics to the idea of positive feedback. The idea underlying positive feedback is that of a change in the world that makes a following change, of similar character but greater magnitude, more likely: examples stand in the capitalist idea of growth, expanding markets, innovations and never-ending progress that have preoccupied economists for long. On the contrary, negative feedback describes situations in which any two consecutive changes have opposite characters, as in perishable products markets, or financial investment, in which any common action results in an opposite result as compared to individual action: if crop producers believe that market prices will be high, a high production for a single individual will imply high earnings from the selling activity for this individual, but generalized high production overfeeds the market and prices, together with individual profits, will collapse.

Using words like *positive* and *negative* doesn't account for *good* and *wrong*. As we will explain in the following subsection, these names are only the extension of derivation signs within any sequence of consecutive changes: positive feedback treats with similarity and positive first derivative (+) and negative feedback deals with opposite effects and negative signed derivative (-). Therefore, positive feedback is not necessarily "positive" and negative feedback is not necessarily "negative". For example, as pointed out by Batten (2004), on a highway, when congestion begins to slow traffic, a downward spiral which will lead to a traffic jam forms; as it begins operating, it functions under positive feedback loops through which traffic is slowed and ultimately brought to a deep basin of an attractor. Every action that slows down traffic - slamming on the brakes, rubbernecking to see an accident - produces more of the same acts. The feedback is positive but its consequence is *negative*: the flow of traffic is decreasing. Let suppose that traffic is completely paralyzed: we are at a point where there is no more change, so no feedback actions, because nothing happens. Acts that could produce change (a clearing in the traffic) tend to be dampened, rather than amplified, as everyone rushes to exploit them and thereby nullifies them. The system has transitioned to negative feedback even if the purpose of actions is a *positive* one. Batten (2004) describes a model of a dynamic world with several punctuated equilibria defined as attractors or steady states that the system is likely to attain in no particular order. Continuing the previous example, one equilibrium is a state of swiftly flowing traffic, and another is a stop-and-go traffic jam. Negative feedback causes a traffic flow in either of these states to remain in it, although we can imagine that the effect of negative feedback is stronger in a traffic jam (that state is a stronger attractor). If the system approaches an attractor, the effect of negative feedback can catch it and keep it there until the process repeats.

We therefore identify a stabilization effect in negative feedback environments and make the assumption that eductive reasoning will perform better in such environments, since its internal coherence is preserved from disturbances and kept in the eductive path. A clarifying point must also be stated about feedback: positive and negative feedback should be described as *effects*, rather than forces. In fact, stabilization works more as an equivalence rather than an oriented implication from one process to another: as a system approaches a steady state, there is not an overarching force of positive feedback that must be overcome. Thus, following the same cited example, when traffic slows to a jam, we needn't look around and wonder what happened to the positive feedback that was driving the change. The positive feedback was an effect of the cars slowing down: once the cars have stopped, we're not going to observe this effect anymore. Therefore, self-enforcement of the rule operates in negative feedback mechanisms. As we will explain later, if we expect eductive reasoning to perform better in negative feedback situations, this is because such a process is inherently driven to equilibrium in a negative feedback situation. In particular, as pointed by Arthur (1989), negative feedback tends to stabilize the economy because any major changes will be offset by the very reactions they generate. Parts of the economy that are resource-based as agriculture are subject to diminishing returns and therefore in a negative feedback configuration. The cobweb environment described by Muth (1961) offers the simplest picture of a negative feedback setting.

4.2. Stability arguments in negative feedback environments

One of the implications of the REH is that expectations are endogenous: they are contingent to the predictions of the relevant economic theory and are self-fulfilling. But as explained earlier, we are interested here in the moment at which the reasoning chain underlined by the REH breaks. When the reasoning process stops, the configuration of the situation can be stable or unstable; in order to understand what exactly happens in a negative feedback environment populated by agents with different cognitive abilities, we have to explore the analytical regularities of economic interactions in such an environment. By the self-fulfilling process, where do expectations drive the situation within a negative feedback setting? Previous assumptions state that the system is drove to stability (to equilibrium). We aim at explaining why and how.

Let us describe a negative feedback environment, following Guesnerie (2004): there is a continuum of agents, each one concerned with his own action and with aggregate data, on which a single agent only has infinitesimal influence. Each individual *i*'s best response depends on the subjective probability distribution on the aggregate data, denoted $p(i)$. To a profile of individual distributions, $P = (\dots p(i)\dots)$, an aggregate situation $B(P)$ is associated, where operator B denotes agent *i*'s best response to the aggregate situation, $B(i, p(i))$. In equilibrium the argument and the realization of the operator B coincide. Let A and A' denote two collections of such point expectations. Negative feedback means that each individual best response function is decreasing in its argument, i.e., given $B(i, A') > B(i, A)$, $A' < A$: to each state of the world, the best-response function gives an opposite new state of the world. Such a formalization has been done by Topkis (1979) and Guesnerie (2004) for positive feedback situations, which they call strategic complementarities situations. By opposition, negative feedback situations are assimilated to strategic substitutabilities situations. As in game theoretical and experimental literature (Van de Velden, 2001; Guesnerie, 2004), both formulations are used to describe situations in which best-response functions are decreasing, we will use in the remaining of this paper several times the denomination "strategic substitutabilities", but we will rather prefer "negative feedback".

Let consider a one-dimensional vector denoted $a > 0$. Strategic substitutabilities or negative feedback assume $(dB /da) < 0$, as in the Muth's (1961) model. As agents involved in the model are producers, the variable a denotes the size of the crop. As price of the crop decreases with a, the size of the supplied crop, $B(a)$, associated with expectations a, decreases with a. The unique equilibrium can be educed as we will explain later, in steps. Guesnerie (2004) gives the following explanation: if some farmers with high costs do not join, then the crop has a maximal size for which the price is low but no so low to prevent a few efficient enough farmers to be willing to produce. But, with the aggregate crop produced only by low cost farmers, the price will be lower than some high threshold, so that some farmers, who a priori wanted to join, drop, so that the price will be higher than previously assumed, so that more slow cost farmers will join etc. The process is more formally described in Guesnerie (1992). The unique equilibrium is globally eductively stable if B has no cycle; it is locally eductively stable *iif* the ratio of the price elasticity of aggregate supply over the price elasticity of aggregate demand is smaller than 1.

Thus $B(a)$ describes the aggregate state of the system[12] as a function of point expectations a. Starting from an initial reference point (which can be one of the borders of the definition interval of *a,* for example the inferior bound 0), the eductive argument adapted to such an environment is the following:

i) *Step* 0: given the negative feedback structure of the environment, $B(0) > a$.
ii) *Step*1: everybody knowing that, and given the negative feedback, the final state of the system a will be such that $a > B \circ B(0)$.
iii) *Step* 2: everybody taking into account the outcome of the previous step (they know and know that the others know), the situation will be such that $a < B \circ B \circ B(0)\dots$

The process continues with any step k taking into account the conclusion of step k-1, and with a alternating positions around compositions of function B. Therefore, if a belongs to a closed interval, there is an unique equilibrium, which is a focal point singled out by the eductive mental process, whereas in the polar case of strategic complementarities or positive feedback, uniqueness of the equilibrium is not guaranteed.

4.3. Intuitive versus reflective beliefs in negative feedback environments

According to the economic psychology literature (Franz, 2003), human thinking (or reasoning) continually performs three operations: scanning data for patterns, storing them in memory, and applying them to make inferences and take decisions. There are situations in which the last operation is reached only through careful, elaborate reasoning; but in some other situations, people are able to spontaneously take decisions. In this paragraph we will briefly remind the functioning of reasoning, as stressed in Sperber (1997)[13], Simon (1965-1997) and Frantz (2003), and that we adapt to negative feedback environments. We focus here on the study of beliefs as structuring

[12] *As* a *is a collection, we can write* B*(*a*)=*a*.*
[13] *This is not at the origin an economic argument, but we consider it as very useful in understanding our research question of eductive abilities, as it incorporates discussion in terms of costs and benefits.*

elements of reasoning. Human mind possesses two kinds of beliefs, intuitive and reflective beliefs. No reflection or specification of particular justifications are needed in order to hold *intuitive* beliefs. But when we are able (need) to draw inferences, to *process* representations (or information) following a scheme or several stages (by using knowledge, common knowledge and reasoning), we hold *reflective* beliefs. When interacting with a situation, an agent is usually able to hold these two kinds of beliefs simultaneously, because simultaneously the human mind has the ability to hold representations as (intuitive) beliefs and has a meta-representational ability (of reflective beliefs). Simon and Gilmartin (1973) stress that intuition is not a process that operates independently of analysis, but by effective decision-making. Reflective beliefs need a validating context or a validating mechanism, whereas intuitive beliefs need what Sperber (1997) calls a creedal context, in the sense that no proof is required when an intuitive belief is activated[14]: Bunge (1962) cites rapid reasoning and Simon (1973) refers to it as subconscious pattern recognition. If an agent holds the intuitive belief *P*, and if *Q* is inferred *spontaneously* from *P*, it is reasonable to attribute to the agent the intuitive belief *Q*. No hierarchy exists between intuitive and reflective beliefs, because intuitive beliefs can turn into reflective beliefs and vice-versa: Simon (1997), for example, defines intuition as analytical complex reasoning (i.e. based on reflective beliefs) frozen into habit and into the capacity for rapid response through recognition of familiar kind of situations, thus as transformed reflective beliefs. But what exists, it is an internal architecture of the human mind, in which all types of beliefs are stored. Talking about storing, and previous considerations on bounded rationality, imply that we consider the representational capacity of an agent to be limited. Intuitive beliefs are likely to necessitate less space than reflective beliefs (in which intermediary statements are equally stored). Sperber (1997) argues that in a language, one may need a concept unavailable in its mental lexicon; one will thus meta-represent this concept with the help of available intuitive concepts: this is the general acceptation of the transition from intuitive to reflective beliefs. As operation repeats, the new meta(constructed) concept may become immediately available: this way a reflective concept turns back into an intuitive one. Paraphrasing Poincaré, "inspiration comes only to the prepared mind". Let remind that the first purpose of our paper is to find when exactly and why is an eductive reasoning likely to stop. We than put forward two assumptions about this precise moment:

i) an agent is likely to stop the eductive reasoning at step *k* if his storing space becomes full at this moment and no space is left for additional introspection steps; the stopping rule is therefore implied by the cognitive (storing and working) capacity of the agent: he stops at step *k* because he fulfil at this step his cognitive constraint (agent has eductive ability *k* or is of *k*-type);

ii) an agent is likely to stop the eductive reasoning at the moment at which it became possible for him to transform the last of the consecutive (reflective) eductive steps into an intuitive statement: an agent stops calculating when it is possible for him to *jump* directly at the (intuitive) equilibrium; in this case, agent's eductive ability is higher than the index *k* of the step at which he stops: the agent stops because it became easy for him to reach the equilibrium point and therefore by doing that he saves efforts and minimizes calculation costs.

In a negative feedback environment, agents beliefs are more likely to be driven by the second assumption. How easily may reflective concepts become intuitive and does a negative feedback environment favour this process? In Sperber (1997), two possible answers and a factual example are suggested. According to a radically empiricist view, no concept is immediately intuitive, but all concepts may become intuitive, provided they are used often enough; according to a radically nativist theory (that the author is closer to), there is an innate range of intuitive concepts, a subset of which becomes actualized in the intuitive repertoire of any given agent. The example lies between these two views: the example is used to identify the moment in which reflective beliefs become intuitive; we adapt it here: imagine agent *i* is holding a visibly "lemon" car; agent *j* observes it and makes statement "what a beautiful car" about it. Agent *i* knows that his car is a clue; agent *j* knows it too; agent *i* knows that agent *j* knows; agent *j* knows that agent *i* knows that agent *j* knows… These iterations are enough for both agents to fully understand the previous agent *j*'s statement. So far, at least 3 iterations have been rapidly made without any particular effort. We are able to communicate at level 3 in day-to-day life within a negative feedback situation and communication works well. This does not hold within a positive feedback situation (if the car *is* beautiful). We therefore conclude that reflection turns into intuition in the neighbourhood of the natural degree of communication (and mutual understanding) in negative feedback environments. We thus expect to find experimental proof of such convergence.

4.4. "Numbers attitude"

As stressed before, we intended to address the eductive reasoning question in contests, i.e. in game

[14] *"All beliefs that are output of perceptual processes are intuitive in a standard psychological sense, and so are all beliefs that are the output of spontaneous or unconscious inferential processes taking intuitive beliefs as premises [...]. When any type of beliefs serves as a premise in a deliberate derivation of further beliefs, the inferential process in which they are involved [leads to] non-intuitive or reflective beliefs". (Sperber, 1997).*

with no pre-play communication (experiments based on the muthian model designed by Guesnerie (1992) and on beauty contest games adapted to negative feedback). In all these experiments, agents are likely to deal with numbers (they may guess numbers, forecast prices or take decisions on quantities). In this paragraph we therefore refer to a study run in experimental psychology by Dehaene and al.(1993), in which the authors explain how people perceive and make use of numbers. The article is concerned with numerical abilities of agents and several regularities are observed in experiments. Among other results, we are interested here in the SNARC (Spatial Numerical Association of Response Codes) concept: numbers are perceived on a scale on which large numbers are associated with one side and small numbers with the other side. The particular direction of the special-numerical association is determined by the direction of writing: small numbers are situated at the left side and high numbers at the right side[15]. What is essential in this numbers architecture is the order: when switching from a number x to another number y the order must be respected, i.e. all numbers between x and y are bypassed and inversely. Let imagine an agent who plays a contest in which numbers are involved in a negative feedback system. At each choice, the system brings the agent in an opposite direction. Let suppose that the eductive reasoning steps (that the agent is supposed to do when making choices) are made in real time rather than in notional time and are thus observable. Suppose that at the first iteration agent switches from x to y, with $x < y$; at the second iteration agent switches back in the direction of x, to z, with $x < z < y$ and then back to t in the direction of y, with $x < z < t < y$ and so on[16], and the process converges to an equilibrium that is likely to be situated between z and t. Therefore, at every switch, the equilibrium number is bypassed; the agents "sees" the equilibrium several times. On the contrary, in a positive feedback situation, equilibrium is outside the sequence of intermediary steps and more and more iterations do not help the agent to better find it. This is the intuition of the argument.

5. Conclusion

The aim of this paper was to introduce the notion of eductive reasoning and to show why it is important to analyse it in a context of negative feedback.

Let imagine a situation of social or economic interaction in which individual agents have to take simultaneous decisions; all fundamentals are known by them. In such a context, people face behavioural uncertainty about their opponents, as they do not hold in their informational set concrete elements about the beliefs that will lead their opponents to take a decision. We can expect people involved together in an economic situation to be willing to reach the equilibrium in that particular situation. Interactive situations are in equilibrium when all agents that participate to the situation are coordinate. In particular, at the equilibrium, agents coordinate their understandings of the situation, and they are compatible: thus coordination passes thought beliefs.

Coordination can either be immediate or reached by some process. When coordination is immediate, this is because the equilibrium is a salient or focal point. If this is not the case, the equilibrium has to be approached in some way, and individuals have to form expectations about it. In economics, several models try to explain the process of expectations formation and coordination by analysing the reasoning mechanism that individuals use. Individuals may either hold perfect expectations about what is going to happened, either not be able of expectations perfection and make mistakes about the future. For example, this is taken into account in the bounded and perfect rationality literature. Evidence from real markets and from laboratory indicates that individuals are likely *not* to be perfectly rational in the sense of the the REH introduced by Muth (1961). But there is also evidence on the fact that individuals *can* be perfect forecasters. We thus addressed in this paper the question of how do people reason when they form expectations and how comes that imperfect rational agents have perfect forecasts.

We therefore stressed that, when rationality is limited, it can be the consequence of involuntary or voluntary restrictions. In that sense, we have to explain the mechanisms of the reasoning process and its stop procedure. Here we focus our attention on *eductive* reasoning, which we defined and put in relation with other types of reasoning and other concepts. Eductive learning is sophisticated reasoning about a situation and about the reasoning of the opponents; therefore, it works through introspection and forecasting about the forecast of the others. It occurs in people mind, in notional time, and thus is different from evolutive procedures run in real time through adaptation. It is based and strongly connected to the notion of common knowledge, as being the internal process which allows increasing the depth of knowledge. It occurs in iterative steps and is likely to stop because of some constraints. Such constraints take into account information processing costs and cognitive abilities. These two elements determine the moment at which the possibly infinite regression of an eductive type of reasoning becomes finite. Agents with low cognitive abilities will involuntary stop this process at the step corresponding to their cognitive constraint. Agents with high cognitive abilities will voluntary stop sophisticating when one addition step will not bring them a sufficient outcome in balance to their effort, and given beliefs that they hold about the reasoning processes run by the opponents. It is more

[15] *For Europeans, for example.*
[16] *We describe here a convergent process.*

interesting for them not to go to the infinitum, but just to stop one step ahead on the others.

Eductive reasoning is complex (difficult) reasoning; therefore, it is important to examine under which conditions this type of reasoning is favoured. In particular, some stabilizing situations may favour the success of eductive reasoning and especially guarantee the fact that at some stopping point in the reasoning process, the equilibrium may still be attained (the system is not too far from the equilibrium). The situation that we take into account is the negative feedback situation; in this context, any deviation is counterbalanced by an opposite one that secures stability in the system. Moreover, this is the context that people face when dealing with price expectations, numbers…etc, because, as psychological research shows, numbers are perceived through oscillatory scanning on a oriented scale. In this context, the moment at which the eductive mechanism stops, may coincide with the moment at which reflective beliefs become intuitive. When this is the case, agents *jump* directly at the equilibrium and (limited) eductive reasoning is successful.

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